DESIGN FORMULAE FOR AERATION DUCTS

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Abstract

Grain may be preserved in aerated stores. The variation of static pressure along an aeration duct has been analysed in a previous publication and some design formulae emanating from that analysis are presented here. Rigorous constraint equations are presented for both suction and blowing ducts and a simple approximate design formula is presented together with a guide as to its application. A numerical example is considered and the results compared with a more detailed analysis.

Introduction

When designing an aeration system for a seed bulk it is important to consider the variation of aeration flow-rate (airflow per unit length of duct) along the length of the duct so that some regions of the seed bulk do not receive too much air and others not enough. Aeration ducts have, in the past, been sized on the basis of rules of thumb such as setting maximum values for, say, the face velocity of air at the duct surface and the maximum longitudinal velocity along the duct (Holman 1960). The present paper gives an approximate formula for selecting the cross-sectional area of aeration ducts together with constraint equations which limit the variation of aeration flow rate to a set fraction of its value at the distal end. The use of the approximate formula is demonstrated by means of a numerical example. This paper should be read in conjunction with Hunter (1986).

Theory

The variation of static pressure in a perforated straight aeration duct was analysed by Hunter (1986). The equation governing the static pressure was found to be:

$$y'' + yy' = \pm y^2 \qquad (1)$$

where the positive sign applies to blowing systems and the negative sign to suction systems. The differentiation is with respect to the parameter z, and y and z have the following interpretation.

z is the non-dimensionalised distance measured from the distal end of the duct and is defined

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$$z = \frac{x}{x_c}$$

x is the actual distance and x_c is called the characteristic length and is defined

$$x_c = \frac{8KA}{P_w f}$$

K is the pressure regain coefficient (taken as 1.5), A is the cross-sectional area of the duct, P_w the wetted perimeter of the duct and f the D'arcy-Weisback friction factor.

y is the non-dimensionalised longitudinal velocity of air in the duct and is defined

$$y = \frac{v}{v_c}$$

where v is the actual longitudinal velocity and v_c is called the characteristic longitudinal velocity and is defined

$$v_c = \frac{\gamma f P_w}{8\rho P_p K^2}$$

γ is called the differential coefficient defined by $\gamma = dp/du$ where p is the static pressure in the duct. ρ is the density of air aind P_p is the perforated perimeter of the duct.

y' is the non-dimensionalised face velocity at the duct surface so that

$$y' = \frac{u}{u_c}.$$

u is the face velocity at the duct surface and u_c is the characteristic value of u and is defined by

$$u_c = \frac{\gamma f^2 P_w^{\ 2}}{64\rho P_p K^3}$$

For a further description the reader is referred to Hunter (1986).

Solutions to the Governing Equation

Solutions to (1) were presented in graphical form in Hunter (1986). A series expansion is also possible and for suction ducts

$$y' = y'_o + \frac{y'^2_o z^2}{2} + \frac{y'^2_o z^3}{3} + \ldots \qquad (2)$$

and for blowing ducts

$$y' = y'_o - \frac{y'^2_o z^2}{2} + \frac{y'^2_o z^3}{3} \ldots \qquad (3)$$

The subscript o refers to the distal end.

Design Equation for Suction Ducts

If the allowable variation in face velocity u expressed as a fraction of u_o is σ, then to the accuracy of (2) we find

$$y'_o = \frac{u_o}{u_c} < \frac{6\sigma}{z^2(3 + 2z)} \qquad (4)$$

where x is taken as equal to the full length of the duct l so that

$$z = \frac{P_w f l}{8KA} \qquad (5)$$

Design Equation for Blowing Ducts

Because of the negative sign in (3) a design equation for blowing ducts cannot be derived directly, however, the following will be found to apply with reference to the graphical solutions to (1) given in Hunter (1986). We have

$$y'_o = \frac{u_o}{u_c} < \frac{6\sigma}{z^2(3-2z)}, \qquad z < 1$$

$$y'_o = \frac{u_o}{u_c} < 6\sigma, \qquad 1 < z < 1.5 \qquad (6)$$

and $$y'_o = \frac{u_o}{u_c} < \frac{6\sigma}{1 + z^2(2z-3)}, \qquad z > 1.5$$

Again z is given by (5).

Approximate Design Formula

To solve equations (4) or (6) so that the design exactly meets the criterion is necessarily an iterative procedure because of the interaction between several of the parameters. It is suggested that the following approximate formula be used and the result confirmed by means of the above equations.

To obtain an approximate formula, the velocity pressure at the fan end of the duct is set equal to the average static pressure Δp multiplied by the fraction σ.

We have

$$\tfrac{1}{2}\rho_a v^2 = \sigma\Delta p \qquad (7)$$

Now $v = Q/A$ where Q is the total air flow from the duct. Also Δp is estimated by RQH/X where R is the first Ergun coefficient (Hunter, 1983) H is the height of the seed bulk and X is the cross-sectional area of the store. (7) becomes

$$\frac{\rho_a}{2A^2} = \frac{\sigma_a RH}{XQ} \;. \qquad (8)$$

Now the specific airflow-rate is given by

$$q = \frac{Q}{\rho_s XH} \qquad (9)$$

and combining (8) and (9) gives

$$\frac{A}{X} = \sqrt{\frac{\rho_a \rho_s q}{2\sigma R}} \;. \qquad (10)$$

The above derivation ignores friction. The second term in each of (2) and (3) derives from static pressure regain and the third term derives from friction. An estimate of the importance of the neglect of the friction term, particularly in the case of suction systems (where friction and static pressure regain are additive) is indicated by the ratio of the third to the second term. The ratio is simply

$$\frac{2z}{3} \text{ or } \frac{P_w fl}{12KA} \;. \qquad (11)$$

If z is less than unity it may be decided to accept the approximate design without referring to the conditions (4) or (6).

Numerical Example

The numerical example is taken from Hunter (1986) and concerns a blowing aeration system in a circular silo which has a straight floor duct across a diameter. The silo is 15 m diameter, the duct 12 m long and the depth of the wheat bulk, 15 m. The density of the air will be taken as 1.2 kg/m^3, the density of the seed bulk as 800 kg/m^3, the specific airflow-rate, 10^{-6}m^3/skg (1ℓ/st). The first and second Ergun coefficients for wheat were taken as 3131 Pa s/m^2 and 10756 Pa s^2/m^3 respectively. The aeration duct of half-round cross-section is to have a flat steel base and a corrugated steel curved surface. The equivalent sand roughness heights ε were taken as 0.00015m and 0.01 m respectively. The airflow requirement was found to be 2.12 m^3/s.

The analysis gave a variation in u/u_o of 0.184 and so setting σ equal to 0.184 in (10) we find for the cross-sectional area of the duct $A = 0.162\ m^2$

Taking the cross-sectional area as 0.162 m, the diameter of the half-round duct d would be 0.642 m compared with 0.5 m used in the original example. To estimate the importance of the friction term we need to calculate the effective friction factor f.

The friction factor is calculated as the weighted average of the values for the curved surface and for the flat base. The formula used to estimate friction factor is that given by Hunter (1986). We have

$$f = \frac{1.326}{\left(\ell n \dfrac{14.88\ A}{P_w \varepsilon}\right)^2} \quad . \tag{12}$$

The wetted perimeter $P_w = d + \pi d/2 = 1.65$ m and so using (12), for the curved surface f = 0.053 and for the flat base f = 0.016.

The perforated perimeter P_p is given by $\pi d/2 = 1.01$ m and the weighted average value for f is given by

$$f = (P_p \times 0.053 + d \times 0.016)/(P_p + d)$$

so $$f = 0.039.$$

From (11) we calculate

$$z = \frac{P_w f \ell}{8KA}$$

so $$z = 0.397.$$

By (11) therefore the friction term is about 26% of the regain term, and by the statement following (11) the design would be accepted without reference to the constraint equations numbered (6). It will be of interest nonetheless to calculate the value of u_o/u_c and compare this with the value given by the constraint equation. We require the value of the parameter γ. Following the procedure described in section 3.1 of Hunter (1986) we have the effective width of the seed bulk cross-section

$$W = X/\ell = 14.7\ m;$$

the depth of seed equivalent to the effect of duct constriction

$$\Delta H = \frac{W}{\pi} \ell n \frac{W}{2P_p} = 9.29\ m\ ,$$

and the pressure difference terms

$$\Delta p_1 = (QR/\ell W)\ (H + \Delta H) = 914\ Pa$$

$$\Delta p_2 = SQ^2/\pi \ell^2 P_p = 106\ Pa$$

and $\Delta p_3 = SQ^2H/\ell^2W^2 = 23$ Pa.

The static pressure across the seed bulk is therefore

$$\Delta p = \Delta p_1 + \Delta p_2 + \Delta p_3 = 1043 \text{ Pa}$$

and the nominal value of u,

$$\bar{u} = Q/\ell P_p = 0.175 \text{ m/s.}$$

$$(\frac{\bar{p}}{\bar{u}}) = \frac{\Delta p}{\bar{u}} = \frac{1043}{0.175} = 5951 \text{ Pas/m} .$$

We have also

$$\phi = (\Delta p_2 + \Delta p_3)/\Delta p_1 = 0.141$$

and $$\gamma = \frac{1+2\phi}{1+\phi} (\frac{\bar{p}}{u}) = 6687 \text{ Pa s/m} .$$

Using the definition given above we now calculate u_c as 0.106 m/s and so using $\bar{u}$ as an estimate of u_o

$$\frac{u_o}{u_c} = 1.65 .$$

Substituting 0.184 for σ and 0.397 for z in (6) we find the constraint value of $u_o/u_c = 3.175$. The constraint equation (6) is therefore well satisfied as suggested earlier.

Discussion

Aeration ducts for stored seed are usually designed on the basis of what has worked in the past rather than on an understanding of what actually takes place within the duct and within the seed bulk.

The assumptions involved in this work are discussed in detail in Hunter (1986). Essentially they are the constancy of the friction factor and proportionality over a limited range between the static pressure in the duct and the face velocity of air at the surface of the duct. It is also assumed that flow of air in the seed bulk is two-dimensional, that is, in planes perpendicular to the axis of the duct. It is considered that this is in fact a limiting assumption and work is proceeding on this aspect of the analysis.

Conclusions

It is important to consider the variation of airflow-rate along an aeration duct to ensure regions of the seed bulk are adequately aerated.

Together with Hunter (1986) the methods discussed herein should give an insight into the mechanism of airflow in ducts under stored seed.

The approximate formula numbered (10) can be used for sizing single ducts and with suitable interpretation of the parameters, for sizing multiple ducts. In each case the confirmatory conditions (4) and (6) are applicable.

References

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