Modelling heat and mass transfer phenomena in bulk stored grains

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Abstract

Markets for durable produce, such as food grains, continue to impose increasingly stringent requirements for high quality commodities This is reflected in demands for zero or very low levels of chemical pesticide residues in grains, very low levels of mycotoxins, high head yield of paddy, a high malting yield of barley and so on. As a result, the design and operation of grain storage systems must be constantly reviewed. Mathematical modelling of the stored grains ecosystem offers the possibility of devising methods of manipulating microclimate within a bulk of grain to ensure that desirable properties of the grain can be preserved during storage. This paper describes some of the recent developments in the formulation of the equations that govern heat and moisture transfer in stored grains. As a result of the work, it has proved possible to calculate from first principles rate processes that occur in bulk-stored grains. A rational method of choosing the simplest, yet accurate, mathematical models is described and results from the work suggest that thermal equilibrium between grains kernels and the intergranular air may be assumed in most commercial applications involving aeration or drying. An analysis of heat and moisture transfer in bulks of high moisture content respiring grains identifies terms that account for the fate of mass and energy associated with the oxidation of the grain substrate. A method of calculating heat and moisture transfer in grain stores of any shape is outlined.

Introduction

Increasingly strict requirements continue to be imposed on the quality of food grains presented onto domestic and international markets. For example, consumers demand low or zero levels of chemical pesticide residues in grains, very low concentrations of mycotoxins, a high milling yield of rice, a high malting yield of barley and so on. In tropical countries, losses resulting from consumption by insects, mites and moulds continue to be unacceptably high. An improved understanding of the stored grains ecosystem is a key to solving these problems. This anthropogenic ecosystem is much simpler than many that occur naturally in that its biological and physical diversity is relatively limited, and the system may be biologically isolated from its wider environment. Furthermore, the microchemical within a grain store can be manipulated by drying or cooling the commodity, or fumigating it with a gas such as phosphene that is toxic to insects.

The devising of strategies to manipulate the microclimates within bulks of stored grains to meet the demands of the market is clearly a multidisciplinary activity that requires inputs from stored products entomologists, toxicologists, mycologists, mathematicians, engineers and so on. This paper outlines some of the recent advances in modelling physical aspects of the stored grains ecosystem, and discusses some areas that require further development. Topics discussed include the establishment of equations that govern heat and mass transfer in bulk stored grains on a firmer foundation than has been traditional. Once the equations have been derived they must be solved subject to boundary conditions that reflect the behaviour of commercial grain stores. For example, fumigant gases released in the headspace of a silo permeate through the grain bulk, and before we can estimate their rate of dispersion we must be able to define the boundary conditions at the interface between the grains and the air in the headspace. It is pointed out in this paper that there is a research focus on solving the equations subject to boundary conditions that reflect the types of grain store found in practice, and this will allow for systems optimisation. Commercial computer software is now available that enables heat, mass and momentum phenomena to be fairly readily simulated. This software awaits exploitation by the community of post-harvest technologists.

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Some Practical Considerations

The local temperature, grain moisture content and composition of the intergranular atmosphere generally dominate biological phenomena that occur in a bulk of stored grains. It is the manipulation of these variables that is available to managers of stored products systems. For example, grain may be cooled by forcing through it cool air. If every region of the grain bulk is to benefit by adopting this strategy the system must be designed so that the air flow rate in every region is sufficiently high to ensure that the grain is cooled sufficiently quickly. At the peripheries of the store where the ambient temperature may prevent the grain from cooling it is desirable to investigate the likelihood of grain drying in these regions. These considerations imply that we must choose the most appropriate location and size of aeration ducts and type of aeration fan required to force air through the grain at the required rate. It is also important to ensure the temperature and humidity of the air used to ventilate the grain are suitable to achieve the required degree of grain cooling. This implies some selection and control of the operation of the aeration system.

Before these commercial considerations can be addressed we require engineering equations that can be used to reliably design storage systems. Such equations are based on the physics of heat and mass transfer and fluid flow. In addition, it is vital that the impact of manipulating the design variables on the biological environment within stored grains can be predicted.

The Fundamental Equations

Most analyses of the heat, mass and momentum transfer phenomena that occur in bulk stored grains are based on the so-called continuum approach, as outlined by Bejan (1984). This terminology arises from the fact that the bulk of grain is considered to be a continuum analogous to a single phase fluid or solid. Properties such as density and composition are deemed to vary continuously in space, whereas in reality they suffer discontinuities at the grain/air surfaces. Such an approach does not account for the fact that heat, mass and momentum transfer in stored grains are more reliably described in terms of phenomena that occur on the length scales of the grain kernels and intergranular pores. For example, heat conduction in the intergranular air is governed by Fourier's law, namely

$$\mathbf{q}_\gamma = k_\gamma \nabla T_\gamma \quad (1)$$

where $\mathbf{q}_\gamma$ is the heat flux, k_γ is the thermal conductivity of air and ∇T_γ is the temperature gradient in the pores. The flow of the air between the grain kernels is governed by the Navier-Stokes equation, which for steady flows may be expressed as

$$\nabla p_\gamma = \rho_\gamma \mathbf{g} + \mu_\gamma \nabla^2 \mathbf{v}_\gamma \quad (2)$$

Equations 1 and 2 are constitutive equations of classical continuum mechanics, and they are well established and of wide applicability. We may also write similar point equations that govern heat and moisture transfer in the grain kernels, and although these may assume the forms of Fourier's and Fick's laws they are only broad descriptions of the phenomena. This is because unlike the intergranular air the grain kernels themselves do not constitute a continuum, but they are a microporous system. However, experimental evidence (Crapiste et al. 1988) suggests that such continuum descriptions are applicable to microporous media such as grain kernels.

Equations 1 and 2 apply to phenomena that occur on the length scales of the grain kernels and intergranular spaces, whereas designers and managers of grain storage systems are concerned with the overall behaviour of a grain store. Engineering design equations need to be expressed in terms of variables such as temperature, grain moisture content, fumigant concentration that vary on the length scale of the macroscopic system. Such variables consist of averages of the temperatures and moisture contents of the grain kernels and air in a representative elementary volume within the grain bulk, as depicted in Figure 1.

Provided the radius of the elementary volume is large compared with the grain kernels and small with respect to the size of the grain bulk the averaged quantities vary smoothly with distance, and their time constants are large compared with those associated with the small length scales. The advantages of the method of volume averaging include:

- Details of the small scale phenomena may be retained in the analysis, and this leads to the possibility of estimating rate coefficients such as the effective diffusivity of moisture through grain bulks (Thorpe et al. 1991a,b) and thermal conductivity (Nozad et al. 1985a,b) from first principles without recourse to experimentation.
- A more detailed analysis of the grain/air/water system enables stored grains technologists to select the simplest mathematical models that are consistent with the required accuracy of calculation (Thorpe and Whitaker 1992a,b).
- The gaining of greater insights into the physical processes that occur in grain bulks. For example, the approach has been used by Thorpe (1993) to identify in greater detail than previously the mass and energy transfers that occur in bulks of respiring grains.
- The possibility of invoking theorems to prove the uniqueness of the engineering design equations derived from the fundamental assumptions.

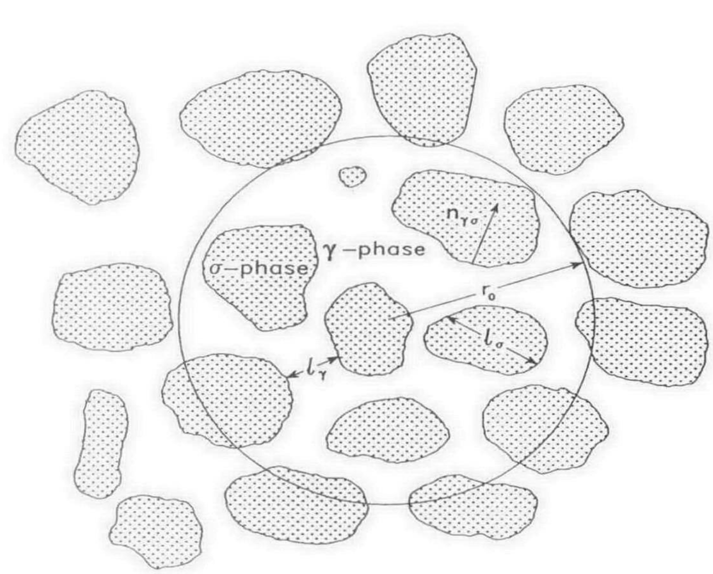

Fig. 1. A representative elementary region of the grain bulk

The Method of Volume Averaging

Carbonell and Whitaker (1984) provide a comprehensive and general description of the method of volume averaging. Its application to specific problems in grain storage engineering are given by Thorpe et al. (1991a,b), Thorpe and Whitaker (1992a,b) and Thorpe (1993). In this paper the emphasis is on the results of the analyses rather than on the methodology. However, it is necessary to present a few key definitions and theorems in order to clarify the discussion. The phase average of a quantity Γ_ω is denoted by $\langle \Gamma_\omega \rangle$ and it is defined by:

$$\langle \Gamma_\omega \rangle = \frac{1}{V} \int_{V_\omega} \Gamma_\omega dV \quad (3)$$

where V_ω is the volume of the ω–phase in representative region. The intrinsic phase average Γ_ω of is defined by:

$$\langle \Gamma_\omega \rangle^\omega = \frac{1}{V_\omega} \int_{V_\omega} \Gamma_\omega dV \quad (4)$$

and if Γ_ω refers to grain moisture content, say, it is the its intrinsic phase average moisture content that is likely to be measured.

When we take the volume average of Equation 1, say, we obtain the average of a Laplacian which is of limited practical use, whereas the Laplacian of an averaged quantity would be a far more useful quantity. The latter may be obtained from the former by successive applications of the volume averaging theorem proved independently by Anderson and Jackson (1967), Marle (1967), Slattery (1967) and Whitaker (1967) and it may be stated as:

$$\langle \nabla \circ \Gamma_\omega \rangle = \nabla \circ \langle \Gamma_\omega \rangle + \frac{1}{V} \int_{A_{\omega\alpha}} \mathbf{n}_{\omega\alpha} \circ \Gamma_\omega dA \quad (5)$$

in which ^{o} represents a tensor multiplication operator of any order, i.e. Γ_ω may be a scalar, vector or tensor of any order. It is useful to make use of the spatial decomposition proposed by Gray (1975), namely

$$\Gamma_{\omega} = \langle \Gamma_{\omega} \rangle^{\omega} + \tilde{\Gamma} \tag{6}$$

The point equations that govern heat, mass and momentum transfer in the grain kernels and intergranular pores may be expressed in terms of volume averaged quantities and spatial deviations from them. Carbonell and Whitaker (1984) have discussed the length-scale constraints that must be satisfied when applying the method of volume averaging. To ensure that averaged quantities vary smoothly with spatial distance the length scales of the grain kernels and intergranular pores, l_γ and l_σ (which are about the same) must be much smaller that tne radius, r_o, of the averaging volume. In addition, the radius of the averaging volume must be much smaller than the length scale, L, over which significant changes in the averaged variables occur. These constraints may be stated as

$$l_\gamma, l_\sigma << r_o << L \tag{7}$$

If the equations that govern the spatial deviations can be formulated and solved we are able to completely define the behaviour of the heat, mass and momentum transfer phenomena that occur in a bulk of grain and other porous media as is evident from the work of Nozad et al. (1985a,b), Ochoa et al. (1986), Thorpe et al. (1991a,b) and Whitaker (1986). This is one of the strengths of the method of volume averaging.

The Effective Diffusion Coefficient in Stored Grains

Moisture diffuses through bulk grains as a result of vapour pressure gradients that arise because of gradients in grain moisture content and/or temperature. An early attempt by Pixton and Griffith (1971) to quantify the rate of moisture diffusion in stored grains made use of the grain moisture content as the driving potential, but this approach lacks generality because the vapour pressure of the interstitial moisture is a non-linear function of grain moisture content and temperature. This results in the effective diffusivity of moisture also being a function of these two variables. Thorpe (1981,1982) recognised that the diffusion of moisture through grains depended almost exclusively on vapour pressure gradients and he used the experimental data of Griffith (1964) and Pixton and Griffith (1971) to calculate the effective diffusivity of moisture as 0.212 that of the diffusivity of moisture vapour in free air. Later, Thorpe et al. (1991a,b) exploited the method of volume averaging to calculate from first principles the effective diffusivity of moisture content in bulk wheat to be 0.24 that of the diffusivity in free air, i.e. in very close agreement with the one obtained experimentally.

The analysis presented by Thorpe et al. (1991a,b) appears to be quite formidable, but it is based on the simply stated equations that govern moisture conservation in the intergranular air and the grain kernels. The equation governing moisture transfer in the intergranular air is:

$$\frac{\partial C_\gamma}{\partial t} = \nabla \cdot \left(\mathscr{D}_\gamma \nabla C_\gamma \right) \qquad \text{in } V_\gamma \tag{8}$$

in which C_γ and $\mathscr{D}_\gamma$ represent the concentration of moisture vapour in tne intergranular air and the molecular diffusion coefficient in of water vapour in air respectively. At the interface of the grain kernels and the intergranular air we have

$$-\mathbf{n}_{\gamma\sigma} \cdot \mathscr{D}_\gamma \nabla C_\gamma = -\mathbf{n}_{\gamma\sigma} \cdot \mathscr{D}_\sigma \nabla C_\sigma \qquad \text{at } A_{\gamma\sigma} \tag{9}$$

where C_σ and $\mathscr{D}_\sigma$ represent the concentration of water in the grain kerneis and the diffusion coefficient of moisture in the solid phase, namely the grain kernels. A unit normal directed from the γ–phase, the intergranular air, to the σ–phase is denoted by $\mathbf{n}_{\gamma\sigma}$. The concentration, C_σ of the moisture on the grain surface is related to that in the air at the surface and the temperature $T_{\gamma\sigma}$ by a general isosteric relationship, thus

$$C_\sigma = \mathscr{F}\left(C_\gamma, T_{\gamma\sigma}\right) \qquad \text{at } A_{\gamma\sigma} \tag{10}$$

Moisture transfer in the grain kernels is governed by

$$\frac{\partial C_\sigma}{\partial t} = \nabla \cdot \left(\mathscr{D}_\sigma \nabla C_\sigma \right) \qquad \text{in } V_\sigma \tag{11}$$

The statement of the mathematical problem is completed by stating the boundary conditions at the periphery of the system shown in figure 1 as

$$C_\gamma = F(\mathbf{r}, t) \qquad \text{at } A_{\gamma\varepsilon} \tag{12}$$

and

$$C_\sigma = G(\mathbf{r}, t) \qquad \text{at } A_{\sigma\varepsilon} \tag{13}$$

where $A_{\gamma\varepsilon}$ and $A_{\sigma\varepsilon}$ are the areas of the γ- and σ-phases at the periphery oi the region under consideration.

Equations 8–13 describe moisture transfer in bulk stored grains, but as they stand they are of little practical value to a grains storage technologist concerned with managing or designing grain stores. This is because they apply only on length scales associated with the grain kernels and the intergranular pores, whereas in practice we are concerned with phenomena that occur on much larger scales, such as the height of a silo. However, the equations contain some information on the rates of mass transfer in the intergranular air and the grain kernels since the diffusion coefficient of moisture through air is widely reported, see Wexler (1965) for example. Furthermore, the diffusion coefficient of moisture in grains has been given by Becker and Sallans (1971) or it may be calculated from drying rate constants such as that presented by O'Callaghan et al. (1971) using the method of Jury (1967). From these constitutive equations of continuum mechanics Thorpe et al. (1991a) show how to apply the volume averaging theorem, expressed as equation 5, and the definition of the spatial deviation, equation 6, to derive a mass transfer equation expressed in terms of a volume averaged concentration, spatial deviations of concentration and empirical rate coefficients. It may be written as

$$\begin{aligned} \left\{ \varepsilon_\sigma \frac{\partial \mathscr{F}}{\partial \{C\}} + \varepsilon_\gamma \right\} \frac{\partial \{C\}}{\partial t} &= \nabla \cdot \left\{ \varepsilon_\gamma D_\gamma \left[\nabla \{C\} + \frac{1}{V_\gamma} \int_{A_{\gamma\sigma}} \mathbf{n}_{\gamma\sigma} \tilde{C}_\gamma dA \right] \right\} \\ &+ \nabla \cdot \left\{ \varepsilon_\sigma D_\sigma \left[\frac{\partial \mathscr{F}}{\partial \{C\}} \nabla \{C\} + \frac{1}{V_\sigma} \int_{A_{\sigma\gamma}} \mathbf{n}_{\sigma\gamma} \tilde{C}_\sigma dA + \frac{\partial \mathscr{F}}{\partial \langle T \rangle} \nabla \langle T \rangle \right] \right\} - \varepsilon_\sigma \frac{\partial \mathscr{F}}{\partial \langle T \rangle} \frac{\partial \langle T \rangle}{\partial t} \end{aligned} \tag{14}$$

in which $\{C\}$ is an equilibrium phase weighted concentration defined as

$$\{C\} = \varepsilon_\sigma \mathscr{F}^{-1}\left(\langle C_\sigma \rangle^\sigma, \langle T \rangle \right) + \varepsilon_\gamma \langle C_\gamma \rangle^\gamma \tag{15}$$

and where $\mathscr{F}^{-1}$ is the inverse of the isosteric equation and it my be defineu as

$$C_\gamma = \mathscr{F}^{-1}\left(C_\sigma, T_{\gamma\sigma}\right) \tag{16}$$

and $\langle T \rangle$ is a phase weighted volume average temperature defined as

$$\langle T\rangle = \varepsilon_\gamma \langle T_\gamma\rangle^\gamma + \varepsilon_\sigma \langle T_\sigma\rangle^\sigma \tag{17}$$

Note that in equation 15 we have used volume averaged arguments of $\mathcal{F}^{-1}$ which Whitaker (1987,1988) has shown to be permissible provided the length scale constraint, 7, is satisfied.

The next task is to find expressions for the spatial deviations $\tilde{C}_\gamma$ and $\tilde{C}_\sigma$ as they occur in a bed of grain. Before this can be done we need to specify the geometry of the grain/air interface. Given that the grain kernels settle in a grain bulk in a random order, this cannot be done precisely, but some average or idealised geometry might be used. Thorpe et al. (1991a,b) assumed that the grains constitute a spatially periodic porous medium, as illustrated in Figure 2. The equations that govern the behaviour of $\tilde{C}_\gamma$ and $\tilde{C}_\sigma$ are difficult to formulate and solve even for the system depicted in Figure 2, and the geometry used is depicted in Figure 3. This has the advantage of realising analytical solutions of the equations that govern $\tilde{C}_\gamma$ and $\tilde{C}_\sigma$, and these may be substituted into equation 14 to arrive at an equation that governs the diffusive moisture transfer of moisture through bulk grains. Thorpe et al. (1991b) show that the effective diffusivity, D_{eff}, of moisture through bulk stored grains is given by the expression

$$D_{eff} = D_\gamma \left\{\frac{2\kappa-(\kappa-1)\varepsilon_\gamma}{2+(\kappa-1)\varepsilon_\gamma}\right\} \Big/ \left\{\varepsilon_\sigma \frac{\partial \mathcal{F}}{\partial\{C\}} + \varepsilon_\gamma\right\} \tag{18}$$

where

$$\kappa = \frac{\partial \mathcal{F}}{\partial\{C\}} \frac{\mathcal{D}_\sigma}{\mathcal{D}_\gamma} \tag{19}$$

Local Mass and Thermal Equilibrium in Bulks of Grain

Optimisation studies of grain storage systems, such as low temperature aeration, often require that many simulations be carried out to help identify the optimum design or operating conditions. For this reason the mathematical models that are used should be as simple, yet as accurate, as possible. This remains important even in this era of readily accessible and rapidly increasing computer power. This is because the models of grain stores apply to increasingly complicated and commercially important geometries, as opposed to the relatively simple heat and moisture transfer models in one-dimensional systems on which the techniques were developed.

Mathematical models of heat and moisture transfer in bulks of grain span a spectrum of complexity. The most complicated identify four independent variables in time and space, namely the temperatures and the moisture contents of the intergranular air and grain kernels. The next lower tier of complexity of models assumes that the air and grains are in local thermal equilibrium, but there is a finite resistance to mass transfer between the grain kernels and the intergranular air. This results in there being three dependent variables at each point in space and time, namely one temperature and a moisture content in each of the two phases present. On the third tier are the simplest models such as those presented by Sutherland et al. (1971) and Ingram (1979) in which both local mass and thermal equilibrium are assumed throughout the bulk. The formulation of such models in one-dimension allows analytical solutions to be obtained. These are useful for illustrating the passage of temperature and moisture waves through bulks of grain, and they are numerically the most efficient to solve, but their analytical solutions are very restrictive in the initial and boundary conditions that can be accommodated.

A rational means must be found for determining the most appropriate model for a given set of conditions, and this is the motivation of the work of Thorpe and Whitaker (1992a,b). The analysis entails introducing macroscopic deviations of temperature and moisture content. Whitaker (1991) defines macroscopic temperature deviations $\hat{T}_\gamma$ and $\hat{T}_\sigma$ as

$$\langle T_\gamma\rangle^\gamma = \langle T\rangle + \hat{T}_\gamma \tag{20}$$

and

$$\langle T_\sigma\rangle^\sigma = \langle T\rangle + \hat{T}_\sigma \tag{21}$$

and when the sorption isotherm of the grains is non-linear Thorpe et al (1991a) define macroscopic concentration deviations as

$$\langle C_\gamma\rangle^\gamma = \{C\} + \hat{C}_\gamma \tag{22}$$

and

$$\langle C_\sigma\rangle^\sigma = \mathcal{F}(\{C\},\langle T\rangle) + \hat{C}_\sigma \tag{23}$$

It can be noted from equations 20 to 23 that when local thermodynamic equilibrium occurs the macroscopic deviations

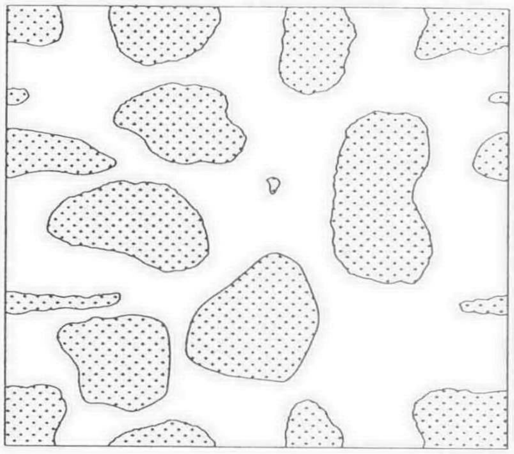

Fig. 2. A representation of a bulk of grains that displays spatial periodicity.

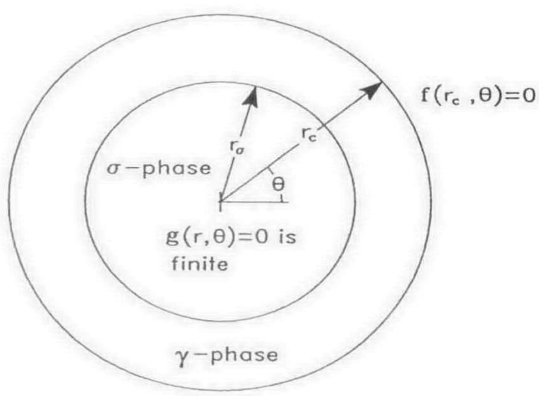

Fig. 3. A simplified unit cell of a spatially periodic porous medium.

are zero Additionally, it is readily deduced that the macroscopic spatial deviations are intimately related to differences in intrinsic averaged quantities in the two phases, i.e.

$$\hat{T}_\gamma = \varepsilon_\sigma \left(\langle T_\gamma \rangle^\gamma - \langle T_\sigma \rangle^\sigma \right) \tag{24}$$

and

$$\hat{C}_\gamma = \varepsilon_\sigma \left[\langle C_\gamma \rangle^\gamma - \mathcal{F}^{-1}\left(\langle C_\sigma \rangle^\sigma , \langle T \rangle \right) \right] \tag{25}$$

The work of Thorpe and Whitaker (1992a) suggests that the thermal energy balance in a ventilated bed of non-respiring grains may be expressed in the terms

$$\begin{aligned} &\left\{ \varepsilon_\gamma (C_\gamma c_p) + \varepsilon_\sigma (C_\sigma c_p) \right\} \frac{\partial \langle T \rangle}{\partial t} + \varepsilon_\gamma (C_\gamma c_p) \langle \mathbf{v}_\gamma \rangle \cdot \nabla \langle T \rangle + \varepsilon_\gamma (C_\gamma c_p) \nabla \cdot \langle \tilde{\mathbf{v}}_\gamma \tilde{T}_\gamma \rangle + \varepsilon_\sigma \langle h_l \dot{m} \rangle \\ &= \nabla \cdot \left[\varepsilon_\gamma k_\gamma \left\{ \nabla \langle T \rangle + \frac{1}{V_\gamma} \int_{A_{\gamma\sigma}} \mathbf{n}_{\gamma\sigma} \tilde{T}_\gamma dA \right\} \right] + \nabla \cdot \left[\varepsilon_\sigma k_\sigma \left\{ \nabla \langle T \rangle + \frac{1}{V_\sigma} \int_{A_{\gamma\sigma}} \mathbf{n}_{\sigma\gamma} \tilde{T}_\sigma dA \right\} \right] \\ &c_{pl} \langle \dot{m} \rangle \hat{T}_\gamma - \varepsilon_\gamma (Cc_p)_\gamma \frac{\partial \hat{T}_\gamma}{\partial t} - \varepsilon_\gamma (C_\gamma c_p) \langle \mathbf{v}_\gamma \rangle \cdot \nabla \hat{T}_\gamma + \nabla \cdot \left(\varepsilon_\gamma k_\gamma \nabla \hat{T}_\sigma \right) \\ &\varepsilon_\sigma (Cc_p)_\sigma \frac{\partial \hat{T}_\sigma}{\partial t} + \nabla \cdot \left(\varepsilon_\sigma k_\sigma \nabla \hat{T}_\sigma \right) \end{aligned} \tag{26}$$

As intimated above, thermodynamic equilibrium is approached as the macroscopic spatial deviations become small. But small compared with what? Whitaker (1991) suggests that they should be considered small compared with the thermal conduction terms in equation 26, which are usually the smallest in magnitude. This implies that in beds of ventilated grains the following constraints must be satisfied

$$\varepsilon_\gamma (Cc_p)_\gamma \frac{\partial \hat{T}_\gamma}{\partial t} + \varepsilon_\sigma (Cc_p)_\sigma \frac{\partial \hat{T}_\sigma}{\partial t} << \nabla \cdot \left[\left(\varepsilon_\gamma k_\gamma + \varepsilon_\sigma k_\sigma \right) \nabla \langle T \rangle \right] \tag{27}$$

$$\varepsilon_\gamma (Cc_p)_\gamma \langle \mathbf{v}_\gamma \rangle^\gamma \cdot \nabla \hat{T}_\gamma << \nabla \cdot \left[\left(\varepsilon_\gamma k_\gamma + \varepsilon_\sigma k_\sigma \right) \nabla \langle T \rangle \right] \tag{28}$$

$$\nabla \cdot \left[\varepsilon_\gamma k_\gamma \nabla \hat{T}_\gamma + \varepsilon_\sigma k_\sigma \nabla \hat{T}_\sigma \right] << \nabla \cdot \left[\left(\varepsilon_\gamma k_\gamma + \varepsilon_\sigma k_\sigma \right) \nabla \langle T \rangle \right] \tag{29}$$

and

$$(c_p)_l \langle \dot{m} \rangle^\gamma \cdot \nabla \hat{T}_\gamma << \nabla \cdot \left[\left(\varepsilon_\gamma k_\gamma + \varepsilon_\sigma k_\sigma \right) \nabla \langle T \rangle \right] \tag{30}$$

Thorpe and Whitaker (1992b) show that the thermal energy equations for each of the solid and fluid phases may be combined to yield estimates of the macroscopic deviations, and that these may be used in the restrictions 27 to 30. Similar reasoning may be applied to develop restrictions that must be satisfied if local mass equilibrium is to occur between the grain kernels and the intergranular air.

As a result of the work of Thorpe and Whitaker (1992a,b) it is possible to show that in practical applications of grain drying or aeration local thermal equilibrium between the kernels and the air may be assumed to exist. This applies to the drying of small grains, such as canola or large tree nuts such as walnuts. The reason for this apparent anomaly is that the width of a drying wave, say, is very narrow when the grains being dried are small, hence temperatures change relatively rapidly but thermal equilibrium is maintained because of the smallness of the particles. When commodities that consist of large particles are being dried the drying wave is wide and temperatures change more slowly in the drying wave, and because of this, thermal equilibrium is maintained between the phases. A consequence of this finding is that mathematical models of heat and moisture transfer processes in bulk stored grains can usually be formulated on the assumption of local thermal equilibrium between the grains and the air. It should be noted that moisture equilibrium cannot generally be assumed, particularly in grain dryers, which is as one would expect. In aeration systems, mass equilibrium is approached as cooling and wetting waves widen when they pass through the grain bulk.

Heat and Mass Transfer in Respiring Bulks of Grains

When grains are stored with moisture contents that result in the relative humidity of the intergranular exceeding about 70% there is a likelihood of mould activity. This gives rise to the production of heat and moisture, and in unaerated grain bulks particularly this may result in moisture migration, thus causing more spoilage of grain. Although some analyses of heat and moisture transfer in respiring bulks of grains have been presented (Thompson, 1971) they fail to fully account for the requirements of mass and energy conservation. For example, when grain respires the carbohydrate substrate of the grain kernels is oxidised. In the process heat and moisture are liberated. As well as these sources of energy and mass, the moisture associated with the disappearing substrate must be accounted for and the surface energy that binds the moisture to the substrate must also be considered. A convenient attack on this problem is described by Thorpe (1993,1994). It commences with statements of the mass and energy conservation equations for the reactive and inert components of the intergranular air and the grain kernels. The boundary conditions at the air/grain kernel interface are stated and the analysis proceeds to develop mass and energy conservation equations expressed in terms of a volume averaged temperature and grain and air moisture contents. The outcomes of the work are expressions for heat and moisture transfer in ventilated beds of respiring grains for two extreme cases - one in which the bed bridges as the grain substrate is consumed and the other in which the bed of grains is deemed to slump. In the bridging bed case the moisture balance may be written

$$\begin{aligned} &\left\{ \varepsilon_\gamma + \varepsilon_\sigma \frac{\partial \mathcal{F}}{\partial \langle (\rho_l)_\gamma \rangle^\gamma} \right\} \frac{\partial \langle (\rho_l)_\gamma \rangle^\gamma}{\partial t} + \left\{ \mathcal{F}\left(\langle (\rho_l)_\gamma , \langle T \rangle \rangle \right) - \langle (\rho_l)_\gamma \rangle^\gamma \right\} \frac{\partial \varepsilon_\sigma}{\partial t} \\ &+ \varepsilon_\sigma \frac{\partial \mathcal{F}}{\partial \langle T \rangle} \frac{\partial \langle T \rangle}{\partial t} + \nabla \cdot \langle (\rho_l)_\gamma \rangle^\gamma \langle \mathbf{v}_\gamma \rangle \\ &= \mathbf{D}_{eff} \cdot \nabla \nabla \langle (\rho_l)^\gamma \rangle + a_v \langle (r_l) \rangle_{\gamma\sigma} \end{aligned} \tag{31}$$

In equation 31, $\langle (\rho_1)_\gamma \rangle^\gamma$ is the concentration of water vapour in the intergranular atmosphere. The concentration of water in the grains, $\langle (\rho_1)_\sigma \rangle^\sigma$ (another way of expressing grain moisture content), is related to the concentration of moisture vapour in the air, $\langle (\rho_1)_\gamma \rangle^\gamma$ and temperature, $\langle T \rangle$, by a general isosteric function expressed in terms of spatially averaged variables, thus

$$\langle (\rho_l)_\sigma \rangle^\sigma = \mathcal{F}\left(\langle (\rho_l)_\gamma \rangle^\gamma , \langle T \rangle \right) \tag{32}$$

The features that distinguish equation 31 from those that govern moisture transfer in grains that are not respiring are the presence of the terms

$$\left\{\mathcal{F}\left(\langle(\rho_1)_\gamma, \langle T\rangle\rangle\right) - \langle(\rho_1)_\gamma\rangle^\gamma\right\}\frac{\partial \varepsilon_\sigma}{\partial t}$$

and $a_v\langle(r_1)\rangle_{\gamma\sigma}$. The former represents changes in moisture concentrations as the grains substrate is consumed by respiration. In particular

$$\mathcal{F}\left(\langle(\rho_1)_\gamma, \langle T\rangle\rangle\right)\frac{\partial \varepsilon_\sigma}{\partial t}$$

accounts for the moisture bound to the disappearing substrate and $\langle(\rho_1)_\gamma\rangle^\gamma \frac{\partial \varepsilon_\sigma}{\partial t}$ arises from the fact that in a bridging bed the intergranular spaces are increasing and they are filled, in part, by moisture vapour. The second term, $a_v\langle(r_1)\rangle_{\gamma\sigma}$, is the rate of production of moisture per unit volume of the grain bulk, and it arises from the oxidation of the grain substrate.

An equation that governs thermal energy conservation in a respiring bulk of grains has been derived by Thorpe (1993) to be

$$\begin{aligned}&\left\{\varepsilon_\gamma \sum_{i=1}^{4}(c_i)_\gamma\langle(\rho_i)_\gamma\rangle^\gamma + \varepsilon_\gamma c_{v2}\langle(\rho_1)_\gamma\rangle^\gamma + \varepsilon_\sigma \sum_{i=1}^{2}(c_i)_\sigma\langle(\rho_i)_\sigma\rangle^\sigma\right\}\frac{\partial\langle T\rangle}{\partial t}\\ &+\varepsilon_\sigma c_{v2} f_2\left(\langle(\rho_1)_\sigma\rangle^\sigma\right)\frac{\partial\langle T\rangle}{\partial t} + \left\{\sum_{i=1}^{4}(c_i)_\gamma\langle(\rho_i)_\gamma\rangle^\gamma\langle \mathbf{v}_\gamma\rangle\right\}\cdot\nabla\langle T\rangle\\ &-\varepsilon_\sigma h_s\frac{\partial\langle(\rho_1)_\sigma\rangle^\sigma}{\partial t} - (\rho_2)_\sigma\frac{\partial \varepsilon_\sigma}{\partial t}\int_0^W h_s\,dW\\ &= \mathrm{K}_{eff}:\nabla\nabla\langle T\rangle + a_v\langle(r_2)_\sigma\rangle_{\gamma\sigma}H_o - a_v\langle(r_1)\rangle_{\gamma\sigma}\langle h_v\rangle^\gamma \end{aligned} \quad (33)$$

The features that distinguish equation 33 from those that do not include respiration are the terms

$$(\rho_2)_\sigma\frac{\partial \varepsilon_\sigma}{\partial t}\int_0^W h_s\,dW$$

and

$$a_v\langle(r_2)_\sigma\rangle_{\gamma\sigma}H_o - a_v\langle(r_1)\rangle_{\gamma\sigma}\langle h_v\rangle^\gamma$$

The first term accounts for the energy associated with the water bound to the grain substrate that disappears as a result of oxidation. The latter group of terms arises from the heat of respiration, and the term $a_v\langle(r_1)\rangle_{\gamma\sigma}\langle h_v\rangle^\gamma$ is associated with the fact that the thermal energy equation is expressed in, amongst other variables, air moisture content and it represents a correction to the heat of respiration which is based on liquid water being formed.

It should be noted that the term

$$\varepsilon_\sigma c_{v2} f_2\left(\langle(\rho_1)_\sigma\rangle^\sigma\right)\frac{\partial\langle T\rangle}{\partial t}$$

should arise in analyses of heat and moisture transfer in both respiring and non-respiring grains, although in this context it appears to have been overlooked by previous authors. Close and Banks (1972) point out its existence in an analysis of coupled heat and mass transfer in silica gel, and it occurs because the integral heat of wetting of the grains is a function of temperature. The constant c_{v2} arises in an expression that relates the latent heat of vaporisation, h_v, of free water and temperature, i.e.

$$h_v = 2502 - c_{v2}T \quad (34)$$

and $f_2\left(\langle(\rho_1)_\sigma\rangle^\sigma\right)$ is a function of the grain moisture content and it may be derived from sorption isotherms as demonstrated by Thorpe et al (1990). The function may be expressed in terms of simple polynomials.

Equations 31 to 33 have been programmed (Thorpe, 1994) for numerical solution by computer, and they were used to assess the likely success of aerating bulk stored grains in the humid tropics.

Solution Procedures

Having established the equations that govern heat and moisture transfer within bulks of grains they must be solved to satisfy the initial and boundary conditions. The initial conditions correspond to the grain temperature and moisture content distributions at the start of the storage period. The boundary conditions on the intergranular air usually imply that the walls and floor of the store are impermeable to gases. Temperatures are usually imposed on the walls and roof of the store and these may be estimated from climatic and solar radiation data, and the floors of grain stores are usually considered to be adiabatic, that is they are perfectly thermally insulated. A procedure for solving the equations that govern heat and mass transfer in three-dimensional bulks of grains is presented by Singh et al (1993b)

A problem of considerable commercial importance is the design of bunker stores for the long term storage of grains. The grains are often stored with initial temperatures exceeding 30°C and as the weather after harvest becomes progressively colder, thermal gradients are set up in the grain bulk and these promote natural convection currents. Warm air from the centre of the bulk rises to the peak where its relative humidity increases, thus increasing the moisture content of the grain in this region. The grain in the vicinity of the peak can become so wet that it is completely spoiled. It is important to investigate design and operational procedures that can overcome these difficulties, and mathematical modelling can be a useful adjunct to this process. Several mathematical models have been developed to predict the rate of migration of heat and moisture resulting from natural convection currents in bunker stored grains. The heat, mass and momentum transfer equations are cast in terms of derived variables, that is a stream function in two-dimensional flows and a vector potential in three-dimensional flows. The resulting equations are solved by discretising them, and solving them on a grid of orthogonal mesh points. In the studies of Nguyen (1987) and Freer et al (1990) the nodes coincide with the physical boundary of the grain store. More recently, Singh and Thorpe (1993a,b) have adopted a numerical scheme whereby the arbitrary physical shapes of grain stores can be transformed (or mapped) into simple shapes such as squares for two-dimensional flows and rectangular parallelepipeds for three-dimensional flows. The basic idea of mapping two-dimensional flow is shown in figure 4. Every point in the physical domain corresponds to a point in the computational domain, in this case a square. The points in the physical domain have (x,y) coordinates and the coordinates in the computational domain are (ξ,η). By the chain rule of differentiation we can express differentials in the physical domain in terms of those in the computational domain, thus

$$\frac{\partial T}{\partial x}=\frac{\partial T}{\partial \xi}\frac{\partial \xi}{\partial x}+\frac{\partial T}{\partial \eta}\frac{\partial \eta}{\partial x} \quad (35)$$

and

$$\frac{\partial T}{\partial y}=\frac{\partial T}{\partial \xi}\frac{\partial \xi}{\partial y}+\frac{\partial T}{\partial \eta}\frac{\partial \eta}{\partial y} \quad (36)$$

Terms such as $\partial\xi/\partial x$ and $\partial\eta/\partial y$ arise from the geometry of the grain store and they can be easily calculated from algebraic formulae or numerically. As Singh and Thorpe (1993a) demonstrate, analogous expressions may be developed for higher derivatives. These expressions may be substituted into the governing heat and moisture transfer equations and solved in the computational domain in (ξ,η) coordinates. The thermal energy balance, equation 33, is thus expressed

$$\begin{aligned}
&\left\{\varepsilon_\gamma \sum_{i=1}^{4}(c_i)_\gamma \langle(\rho_i)_\gamma\rangle^\gamma + \varepsilon_\gamma c_{v2}\langle(\rho_1)_\gamma\rangle^\gamma + \varepsilon_\sigma \sum_{i=1}^{2}(c_i)_\sigma \langle(\rho_i)_\sigma\rangle^\sigma\right\}\frac{\partial\langle T\rangle}{\partial t}\\
&+\varepsilon_\sigma c_{v2} f_2\left(\langle(\rho_1)_\sigma\rangle^\sigma\right)\frac{\partial\langle T\rangle}{\partial t}+\sum_{i=1}^{4}(c_i)_\gamma\langle(\rho_i)_\gamma\rangle^\gamma\left[u_\gamma\left\{\alpha_1\frac{\partial T}{\partial \xi}+\alpha_2\frac{\partial T}{\partial \eta}\right\}\right.\\
&\left.+v_\gamma\left\{\alpha_3\frac{\partial T}{\partial \xi}+\alpha_4\frac{\partial T}{\partial \eta}\right\}\right]-\varepsilon_\sigma h_s\frac{\partial\langle(\rho_1)_\sigma\rangle^\sigma}{\partial t}-(\rho_2)_\sigma\frac{\partial \varepsilon_\sigma}{\partial t}\int_0^W h_s dW\\
&=\beta_1\frac{\partial^2 T}{\partial \xi^2}+\beta_2\frac{\partial^2 T}{\partial \eta^2}+\beta_3\frac{\partial^2 T}{\partial \xi\partial \eta}+\beta_4\frac{\partial T}{\partial \xi}+\beta_5\frac{\partial T}{\partial \eta}+a_v\langle(r_2)_\sigma\rangle_{\gamma\sigma}H_o-a_v\langle(r_1)\rangle_{\gamma\sigma}\langle h_v\rangle^\gamma
\end{aligned} \quad (37)$$

in which

$$\alpha_1=\frac{\partial \xi}{\partial x},\ \alpha_2=\frac{\partial \eta}{\partial x},\ \alpha_3=\frac{\partial \xi}{\partial y},\ \alpha_4=\frac{\partial \xi}{\partial y} \quad (38a)$$

$$\beta_1=\alpha_1^2+\alpha_3^2, \beta_2=\alpha_2^2+\alpha_4^2, \beta_3=2(\alpha_1\alpha_2+\alpha_3\alpha_4), \quad (38b)$$

$$\beta_4=2\left(\frac{\partial^2\xi}{\partial x^2}+\frac{\partial^2\xi}{\partial y^2}\right),\ \beta_5=2\left(\frac{\partial^2\eta}{\partial x^2}+\frac{\partial^2\eta}{\partial y^2}\right) \quad (38c)$$

The important feature of equation 37 is that it can be solved using the same procedures as Nguyen (1987) and Freer et al. (1990), but the shape of the grain store can be arbitrary.

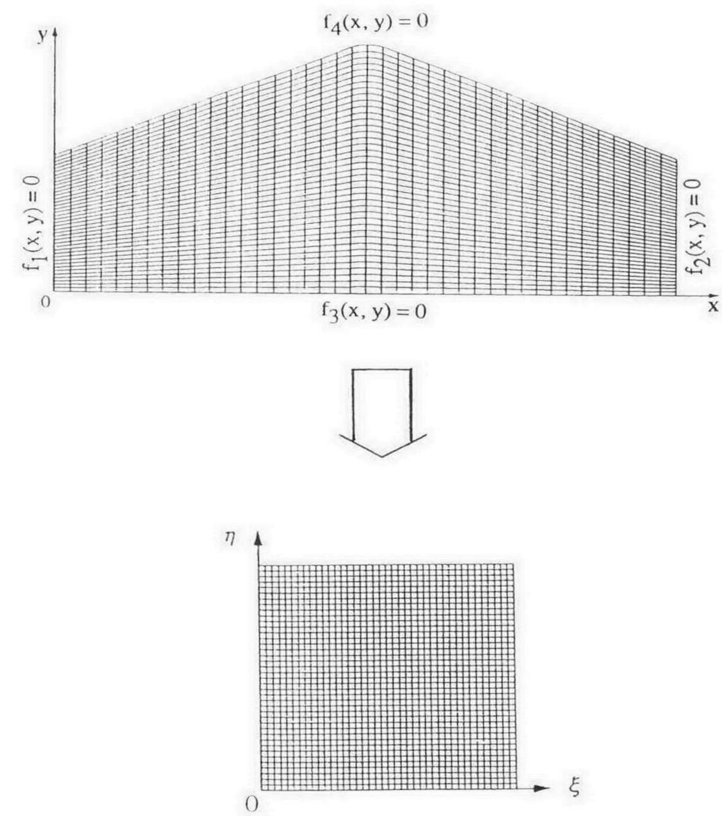

Fig. 15. Mapping the cross-section of a bulk of grain into an orthogonal computational domain.

Future Developments

Modelling transport phenomena in the headspace

To date, mathematical models of grain stores have not treated the heat, mass and momentum transfer phenomena that occur in the headspace of grain stored in the same detail as the phenomena in the grain bulk. Reasons for this are the complex nature of the turbulent fluid flow in the headspace, and the as yet incompletely resolved physics of the interaction of the air in the headspace and the grain bulk. Singh et al. (1993a) and Singh et al. (1994) have studied laminar flows in fluids overlaying porous media using two formulations of the boundary conditions at the fluid/porous medium interface, namely the Beavers-Joseph (Beavers and Joseph 1967) conditions and the Brinkman (1947) conditions. This research needs to be extended to account for the turbulent nature of the flow in the headspaces of grain stores, and the logical next step is to exploit the methods used by Gatheri et al. (1993) who have developed mathematical descriptions of turbulent flows in enclosures.

The use of commercial software packages

Fundamental studies of computational fluid dynamics carried out some two decades ago are now giving rise to powerful and fairly user-friendly commercial software packages. The packages are used in industries such as aerospace, chemical processing, automotive design and so on to predict the performance of components such as cooling towers, turbomachinery, furnaces, air conditioning systems and quite recently in grain storage applications. The packages, such as PHOENICS produced by CHAM (Proprietary information), can be used to discretise the differential equations that govern heat, mass and momentum transfer in stored grains, and in particular the coordinate system can be manipulated to fit the shape a grain store, say. Such packages clearly offer considerable scope for grain storage technologists to solve very complicated problems.

Acknowledgment

The author is grateful to Associate Professor I. D. G. Mackie, Head, Department of Civil and Building Engineering, for encouraging this work to be carried out.

Nomenclature

a_v	Area per unit volume of bed, 1/m
c_i	Specific heat of *i*th species, J/kgK.
c_{v2}	Constant in expression for latent heat of evaopration of free water, J/kgK
$\{C\}$	Phase weighted equilibrium concentration, kg/m^3
C_ω	Concentration of species ω-phase, kg/m^3
$\mathcal{B}_\omega$	Binary diffusion coefficient, m^2/s
cw1,...,cw5	Empirical constants in the isotherm equation.
$\mathbf{D}_{eff}$	Effective diffusivity, m^2/s
$\mathcal{F}$	Sorption isotherm
f_2	An empirical function of moisture content.
H_o	Heat of oxidation of grain substrate, J/kg
h_s	Heat of sorption of moisture on grain substrate, J/kg
h_v	Latent keat of vaporization of water, J/kg.
$\mathbf{K}_{eff}$	Effective thermal dispersivity tensor, J/kg/K
k_ω	Thermal conductivity of ω-phase, J/m/K.
l_ω	Length scale of ω-phase, m.
L	Length scale of macroscopic system, m.
$\dot{m}$	Rate of moisture transfer, kg/s/m^3
$\mathbf{n}_{\alpha\omega}$	Unit normal in the direction α–ω
p_o	Grain-specific constant in isotherm equation, Pa.
p_s	Saturation pressure of free water, Pa.
p_ω	Pressure in ω-phase, Pa.
$\mathbf{q}_\omega$	Heat flux through ω-phase, W/m^2.
r_i	Rate of production or disappearance of *i*th species per unit area, kg/s/m^2
r_o	Radius of representative elementary volume, m.
$\mathbf{r}$	A position vector, m.
t	Time, s
$\langle T \rangle$	Phase weighted average temperature, K.
T^o	Reference temperature, K.
T_ω	Temperature of ω-phase, K.
u	Horizontal component of velocity, m/s
$\mathbf{u}_i$	Diffusion velocity of *i*th species, m/s.
v	Vertical component of velocity, m/s.
$\mathbf{v}_i$	Velocity of *i*th species, m/s.
V	Volume of representative elementary region, m^3.
V_ω	Volume of ω-phase in representative elementary region, m^3.
$\mathbf{w}$	Velocity of surface of grain kernels being consumed by respiration, m/s
W	Moisture content of grain (dry basis), kg/kg.
x,y	Cartesian coordinates, m.
Greek symbols	
α	Refers to α-phase
$\alpha_1,\ldots,\alpha_4$	Mesh transformation functions.
$\beta_1,\ldots,\beta_5$	Mesh transformation functions.
γ	Refers to γ-phase
Γ_ω	A quantity defined in the ω-phase.
$\langle \Gamma_\omega \rangle$	Superficial volume average of Γ_ω.
$\langle \Gamma_\omega \rangle^\omega$	Intrinsic volume average of Γ_ω.
$\langle \Gamma \rangle_{\alpha\omega}$	Area average of Γ at the α–ω surface.
$\tilde{\Gamma}_\omega$	Spatial deviation of Γ_ω from $\langle \Gamma_\omega \rangle^\omega$.
$\hat{\Gamma}$	Macroscopic deviation of $\langle \Gamma_\omega \rangle^\omega$ from $\langle \Gamma \rangle$
ε_ω	Void fraction of ω-phase.
κ	Defined by equation 19.
μ	Viscosity, kg/m/s.
ξ,η	Coordinates of the computational domain.
ρ_i	Density of *i*th species, kg/m^3.
σ	Refers to σ-phase
ω	Refers to ω-phase

References

Anderson, T.B. and Jackson, R. 1967. A fluid mechanical description of fluidised beds. Industrial Engineeing and Chemistry Fundamentals, 6, 527–539.

Beavers G. and Joseph, D.D. 1967. Boundary conditions at a naturally permeable wall. Journal of Fluid Mechanics, 110, 197–207.

Becker, H.A. and Sallans, H.R. 1971. Drying wheat in a spouted bed. On the continuous moisture diffusion controoled drying of solid particles in a well-mixed, isothermal bed. Chemical Engineering Society, 13, 97–112.

Bejan, A. 1984. Convection heat transfer. New York, Wiley.

Brinkman, H.C. 1947. A calculation of the viscous force exerted by a flowing fluid on a dense swarm of particles. Applied Science Research, A1, 27–34.

Carbonell, R.G. and Whitaker, S. 1984. Heat and mass transfer in porous media In: Bear, J. and Corapciolglu, M. Y., eds., Mechanics of fluids in porous media, Nijhof, Brussels, 121–198.

CHAM (Proprietary information) What is PHOENICS? Concentration Heat and Momentum Limited, London, UK.

Close, D.J. and Banks, P. J. 1972. Coupled equilibrium heat and single adsorbate transfer in fluid flow through a porous medium – II. Predictions for silica gel-air using characteristic charts. Journal of Chemical Engineering Science, 27(5),1157–1169.

Crapiste, G.H., Whitaker, S. and Rotstein, E. 1988. Drying of cellular material I. A mass transfer theory. Chemical Engineering Science, 43, 2919–2928.

Freer, M.W., Siebenmorgan, T.J., Couvillion, R.J. and Loewer, O.J. 1990. Modeling temperature and moisture content changes in bunker-stored rice. Transactions ASAE, 33, 211–220.

Gatheri, F.K., Reizes, J.A., Leonardi, E. and de Vahl Davis, G. 1993. The use of variable false transient parameters for the solution of natural convection problems. Fifth Australasian Heat and Mass Transfer Conference, Brisbane, 6–9th December, 1993.

Gray, W.G. 1975. A derivation of the equation for multiphase transport. Chemical Engineering Science, 30, 229–233.

Griffith, H.J. 1964. Bulk storage of grain: a summary of factors governing control of deterioration. Melbourne, Australia, CSIRO Division of Mechanical Engineering, Report ED8.

Ingram, G.W. 1979. Solution of grain cooling and drying problems. Journal of Agricultural Engineering Research, 24, 219–232.

Jury, S.H. 1967. An improved vesion of the state equation for molecular diffusion in a dispersed phase. AIChEJournal, 13,1124–1126.

Marle, C.M. 1967. Ecoulements monophasiques en milieu poreux. Rev. Fr. Petr., 2, 327–356.

Nguyen, T.V. 1987. Natural convection in stored grains—a simulation study. Drying Technology, 5, 541–600.

Nozad, I, Carbonnel, R.G. and Whitaker, S. 1985a. Heat conduction in multiphase systems— I. Theory and experiment for two phase systems. Chemical Engineering Science, 40, 843–855.

Nozad, I, Carbonnel, R.G. and Whitaker, S. 1985b. Heat conduction in multiphase systems—II. Experimental method and results for three phase systems. Chemical Engineering Science, 40, 857–863.

O'Callaghan, J.R., Menzies, D.J. and Bailey, P.H. 1971. Digital simulation of agricultural drier performance. Journal of Agricultural Engineering Research, 16, 223–244.

Ochoa, J.A. Stroeve, P. and Whitaker, S. 1986. Diffusion and reaction in cellular media. Chemical Engineering Science, 40, 943–855.

Pixton, S.W. and Griffith, H.J. 1971. Diffusion of moisture through grain. Journal of Stored Products Research, 7, 133–152.

Singh, A.K and Thorpe, G.R. 1993a. A solution procedure for three-dimensional free convective flows in peaked bulks of grain. Journal of Stored Products Research, 28 (3), 221–235.

Singh, A.K and Thorpe, G.R. 1993b. Application of a grid generation technique to the numerical modelling of heat and moisture movement in peaked bulks of grains. Journal of Food Processing Engineering, 16 (2), 127–145.

Singh, A.K., Leonardi, E and Thorpe, G.R. 1993a. Three-dimensional free convection in a confined fluid overlying a porous layer. Journal of Heat Transfer, 115 (3), 631–638..

Singh, A.K., Leonardi, E and Thorpe, G.R. 1993b. A solution procedure for the equations that govern three-dimensional free convection in bulk stored grains. Transactions ASAE, 36(4), 1159–1173.

Singh, A.K., Moore, G.A. and Thorpe, G.R. 1994. Effect of the ratio of the depths of fluid and porous layers on free convective flows in tall rectanglar cavities. 12th National Heat and Mass Transfer Conference, January 5–7, Bombay, India.

Slattery, J.M. 1967. Flow of viscoelastice fluids through porous media. AIChE Journal, 13,1066–1071.

Sutherland, J.W., Banks, P.J. and Griffith, H.J. 1971. Equilibrium heat and moisture transfer in air flow through grain. Journal of Agricultural Engineering Research, 16, 368–386.

Thompson, T.L. 1972. Temporary storage of high-moisture sheeled corn using continuous aeration. Transactions ASAE, 15, 333–337.

Thorpe, G.R. 1981. Moisture diffusion through bulk stored grain. Journal of Stored Products Research, 17, 39–42.

Thorpe, G.R. 1982. Moisture diffusion through bulk grain subjected to a temperature gradient. Journal of Stored Products Research, 18, 9–12.

Thorpe, G.R. 1993. Heat and mass transfer in ventilated bulks of respiring porous hygroscopic media. Fifth Australasian Conference on Heat and Mass Transfer, December 6–9, Brisbane, 1993.

Thorpe, G.R. 1994. Heat and moisture transfer in ventilated bulks of respiring grains—A theoretical analysis and technological application. Report, Department of Civil and Building Engineering, Victoria University of Technology, Melbourne, Australia.

Thorpe, G.R. and Whitaker, S. 1992a. Local mass and thermal equilibria in ventilated grain bulks. Part I The development of heat and mass conservation equations. Journal of Stored Products Research, 28, 15–27.

Thorpe, G.R. and Whitaker, S. 1992b. Local mass and thermal equilibria in ventilated grain bulks. Part II The development of constraints. Journal of Stored Products Research, 28, 29–54.

Thorpe, G.R., Ochoa, J.A. and Whitaker, S. 1991a. The diffusion of moisture in food grains. I The development of a mass transfer equation. Journal of Stored Products Research, 27, 1–9.

Thorpe, G.R., Ochoa, J.A. and Whitaker, S. 1991b. The diffusion of moisture in food grains. II Estimation of the effective thermal diffusivity. Journal of Stored Products Research, 27, 11–30.

Thorpe, G.R., Stokes, A.N. and Wilson, S.G. 1990. The integral heats of wetting of food grains, Journal of Agricultural Engineering Research, 46, 71–76.

Wexler, A. 1965. Humidity and moisture. In: Wexler, A., ed., Measurement and Control in Science and Industry, 1, Reinhold, New York.

Whitaker, S. 1967. Diffusion and dispersion in porous media. AIChE Journal, 13, 420–427.

Whitaker, S. 1986. Flow in porous media I: A theoretical derivation of Darcy's law, Transport in Porous Media, 1, 3–25.

Whitaker, S. 1987. The role of the volume averaged temperature in the analysis of non–isothermal, multiphase transport phenomena. Chemical Engineering Communication, 58, 171–183.

Whitaker, S. 1988. Coments and corrections concerning the volume-averaged temperature and its spatial deviation. Chemical Engineering Communication, 70, 15–18.

Whitaker, S. 1991. Some improved estimates for the principle of local thermal equilibrium. Industrial Engineering and Chemistry Research 30, 983–997.